edoceralph

Quotientenregel:

$$f(x) = \frac{u(x)}{v(x)}$$

$$f'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v^2(x)}$$

Aufgabe 1

$$f(x) = \frac{2x}{x+1}$$

Lösung

1. Ableitung

$u(x) = 2x$ $\quad$ $u'(x) = 2$

$v(x) = x+1$ $\quad$ $v'(x) = 1$

$$f'(x) = \frac{2 \cdot (x+1) - 2x \cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

2. Ableitung

$u(x) = 2$ $\quad$ $u'(x) = 0$

$v(x) = (x+1)^2$ $\quad$ $v'(x) = 2(x+1)$

$$f'(x) = \frac{0 \cdot (x+1)^2 - 2 \cdot 2(x+1)}{(x+1)^4} = \frac{-4\cancel{(x+1)}}{(x+1)^{\cancel{4}3}} = \frac{-4}{(x+1)^3}$$

Aufgabe 2

$$f(x) = \frac{4x^2}{2x-2}$$

Lösung

1. Ableitung

$u(x) = 4x^2$ $\quad\quad$ $u`(x) = 8x$

$v(x) = 2x-2$ $\quad\quad$ $v`(x) = 2$

$$f`(x) = \frac{8x\cdot(2x-2)-4x^2\cdot 2}{(2x-2)^2} = \frac{8x^2-16x}{(2x-2)^2}$$

2. Ableitung

$u(x) = 8x^2-16x$ $\quad\quad$ $u`(x) = 16x-16$

$v(x) = (2x-2)^2$ $\quad\quad$ $v`(x) = 2(2x-2)\cdot 2$

$$f`(x) = \frac{(16x-16)\cdot(2x-2)^2-(8x^2-16x)\cdot 4(2x-2)}{(2x-2)^4}$$

$$= \frac{\cancel{(2x-2)}\cdot[(16x-16)\cdot(2x-2) - 4\left(8x^2-16x\right)]}{(2x-2)^{\cancel{4}3}}$$

$$= \frac{(16x-16)\cdot(2x-2) - 4(8x^2-16x)}{(2x-2)^3}$$

$$= \frac{32x^2-64x+32 - 32x^2+64x)}{(2x-2)^3}$$

$$= \frac{32}{(2x-2)^3}$$

Aufgabe 3

$f(x) = \frac{x}{\sin(x)}$

Lösung

1. Ableitung

$u(x) = x \qquad u'(x) = 1$

$v(x) = \sin(x) \qquad v'(x) = \cos(x)$

$$f'(x) = \frac{1 \cdot \sin(x) - x \cdot \cos(x)}{\sin^2(x)}$$

2. Ableitung

$u(x) = \sin(x) - x\cos(x) \qquad u'(x) = \cos(x) - \cos(x) + x\sin(x)$

$= x\sin(x)$

$v(x) = \sin^2(x) \qquad v'(x) = 2\sin(x) \cdot \cos(x)$

$$f'(x) = \frac{x\sin(x) \cdot \sin^2(x) - (\sin(x) - x\cos(x)) \cdot (2\sin(x)\cos(x))}{\sin^4(x)}$$

$$= \frac{\cancel{\sin(x)} \cdot [x\sin^2(x) - (2\cos(x) \cdot (\sin(x) - x\cos(x))]}{\sin^{\cancel{4}3}(x)}$$

$$= \frac{x\sin^2(x) - (2\cos(x) \cdot (\sin(x) - x\cos(x))}{\sin^3(x)}$$

Aufgabe 4

$f(x) = \frac{x^2}{e^x}$

Lösung

1. Ableitung

$u(x) = x^2$ $u'(x) = 2x$

$v(x) = e^x$ $v'(x) = e^x$

$$f'(x) = \frac{2x \cdot e^x - x^2 \cdot e^x}{e^{2x}} = \frac{\cancel{e^x} \cdot (2x - x^2)}{e^{\cancel{2}x}} = \frac{(2x - x^2)}{e^x}$$

2. Ableitung

$u(x) = 2x - x^2$ $u'(x) = 2 - 2x$

$v(x) = e^x$ $v'(x) = e^x$

$$f'(x) = \frac{(2 - 2x) \cdot e^x - (2x - x^2) \cdot e^x}{e^{2x}}$$

$$= \frac{\cancel{e^x} \cdot (x^2 - 4x - 2)}{e^{\cancel{2}x}}$$

$$= \frac{x^2 - 4x - 2}{e^x}$$

Aufgabe 5

$$f(x) = \frac{x^2}{3x^2 + x - 1}$$

Lösung

1. Ableitung

$u(x) = x^2$ $\qquad$ $u'(x) = 2x$

$v(x) = 3x^2 + x - 1$ $\qquad$ $v'(x) = 6x + 1$

$$f'(x) = \frac{2x\cdot\left(3x^2+x-1\right) - x^2\cdot(6x+1)}{(3x^2+x-1)^2} = \frac{x^2-2x}{(3x^2+x-1)^2}$$

2. Ableitung

$u(x) = x^2 - 2x$ $\qquad$ $u'(x) = 2x - 2$

$v(x) = (3x^2 + x - 1)^2$ $\qquad$ $v'(x) = 2(3x^2 + x - 1)(6x + 1)$

$$f'(x) = \frac{(2x-2)\cdot\left(3x^2+x-1\right)^2-(x^2-2x)\cdot2(3x^2+x-1)(6x+1)}{(3x^2+x-1)^4}$$

$$= \frac{\cancel{(3x^2+x-1)}\cdot[(2x-2)\cdot\left(3x^2+x-1\right)^2-2\left(x^2-2x\right)(6x+1)]}{(3x^2+x-1)^{\cancel{4}3}}$$

$$= \frac{-6x^3+18x^2+2}{(3x^2+x-1)^3}$$