

edoceralph

Formansatz:

Aufgabe 1

$$f(x) = (4x+5) \cdot e^x$$

$$F(x) = (ax+b) \cdot e^x \quad \Rightarrow \quad F'(x) = a \cdot e^x + (ax+b) \cdot e^x$$
$$F'(x) = (ax + (a+b)) \cdot e^x$$

Koeffizientenvergleich:

$$4 = a$$

$$5 = a + b \Rightarrow b = 1$$

$$\Rightarrow F(x) = (4x+1) \cdot e^x$$

Probe:

$$F'(x) = 4 \cdot e^x + (4x+1) \cdot e^x = (4x+5) \cdot e^x = f(x)$$

Aufgabe 2

$$f(x) = (4x-1) \cdot e^{3x}$$

$$F(x) = (ax+b) \cdot e^{3x} \quad \Rightarrow \quad F'(x) = a \cdot e^{3x} + 3(ax+b) \cdot e^{3x}$$
$$F'(x) = (ax + (a+b)) \cdot e^x$$

Koeffizientenvergleich:

$$4 = 3a \quad \Rightarrow \quad a = \frac{4}{3}$$

$$-1 = a + 3b \quad \Rightarrow \quad b = -\frac{7}{9}$$

$$\Rightarrow F(x) = \left(\frac{4}{3}x - \frac{7}{9}\right) \cdot e^{3x}$$

Probe:

$$F'(x) = \frac{4}{3} \cdot e^{3x} + 3\left(\frac{4}{3}x - \frac{7}{9}\right) \cdot e^{3x} = (4x-1) \cdot e^{3x} = f(x)$$

Aufgabe 3

$$f(x) = (4x^2 - 4) \cdot e^x = (4x^2 + 0x - 4) \cdot e^x$$

$$F(x) = (ax^2 + bx + c) \cdot e^x \Rightarrow F'(x) = (2ax + b) \cdot e^x + (ax^2 + bx + c) \cdot e^x$$

$$F'(x) = (ax^2 + (2a+b)x + (b+c)) \cdot e^x$$

Koeffizientenvergleich:

$$4 = a$$

$$0 = 2a + b \Rightarrow b = -8$$

$$-4 = b + c \Rightarrow c = 4$$

$$\Rightarrow F(x) = (4x^2 - 8x + 4) \cdot e^x$$

Probe:

$$F'(x) = (8x - 8) \cdot e^x + (4x^2 - 8x + 4) \cdot e^x = (4x^2 - 4) \cdot e^x = f(x)$$

Aufgabe 4

$$f(x) = (x^2 - 1) \cdot e^{-x} = (x^2 + 0x - 1) \cdot e^{-x}$$

$$F(x) = (ax^2 + bx + c) \cdot e^{-x} \Rightarrow F'(x) = (2ax + b) \cdot e^{-x} - (ax^2 + bx + c) \cdot e^{-x}$$

$$F'(x) = (-ax^2 + (2a-b)x + (b-c)) \cdot e^{-x}$$

Koeffizientenvergleich:

$$1 = -a \Rightarrow a = -1$$

$$0 = 2a - b \Rightarrow b = -2$$

$$-1 = b - c \Rightarrow c = -1$$

$$\Rightarrow F(x) = (-x^2 - 2x - 1) \cdot e^{-x}$$

Probe:

$$F'(x) = (-2x - 2) \cdot e^{-x} - (-x^2 - 2x - 1) \cdot e^{-x} = (x^2 - 1) \cdot e^{-x} = f(x)$$

Aufgabe 5

$$f(x) = (x^3 - 2x^2 + 4) \cdot e^x = (x^3 - 2x^2 + 0x + 4) \cdot e^x$$

$$F(x) = (ax^3 + bx^2 + cx + d) \cdot e^x$$

$$\Rightarrow F'(x) = (3ax^2 + 2bx + c) \cdot e^x + (ax^3 + bx^2 + cx + d) \cdot e^x$$

$$F'(x) = (ax^3 + (3a+b)x^2 + (2b+c)x + (c+d)) \cdot e^x$$

Koeffizientenvergleich:

$$1 = a$$

$$-2 = 3a + b \quad \Rightarrow b = -5$$

$$0 = 2b + c \quad \Rightarrow c = 10$$

$$4 = c + d \quad \Rightarrow d = -6$$

$$\Rightarrow F(x) = (x^3 - 5x^2 + 10x - 6) \cdot e^x$$

Probe:

$$F'(x) = (3x^2 - 10x + 10) \cdot e^x + (x^3 - 5x^2 + 10x - 6) \cdot e^x = (x^3 - 2x^2 + 4) \cdot e^x = f(x)$$