edoceralph

Uneigentliche Integrale:

Aufgabe 1

$f(x) = \int_1^{\infty} \frac{1}{x^2}\, dx$

$f(x) = \lim\limits_{b\to\infty} \left(\int_1^b \frac{1}{x^2}\, dx\right) = \lim\limits_{b\to\infty} \left(\left[-\frac{1}{x}\right]_1^b\right) = \lim\limits_{b\to\infty} \left(\left(-\frac{1}{b}\right) - \left(-\frac{1}{1}\right)\right)$

$= \lim\limits_{b\to\infty} \left(1 - \frac{1}{b}\right) = 1$

Aufgabe 2

$f(x) = \int_0^1 \frac{2}{\sqrt{x}}\, dx$

$f(x) = \int_0^1 \frac{2}{\sqrt{x}}\, dx = \left[4\sqrt{x}\right]_0^1 = \left(4\sqrt{1}\right) - \left(4\sqrt{0}\right) = 4$

Aufgabe 3

$f(x) = \int_{-\infty}^0 e^x\, dx$

$f(x) = \lim\limits_{b\to-\infty} \left(\int_b^0 e^x\, dx\right) = \lim\limits_{b\to-\infty} \left(\left[e^x\right]_b^0\right) = \lim\limits_{b\to-\infty} \left(\left(e^0\right) - \left(e^b\right)\right)$

$= \lim\limits_{b\to-\infty} \left(1 - e^b\right) = 1 - 0 = 1$

Aufgabe 4

$f(x) = \int_0^{\infty} x \cdot e^{-x^2}\, dx$

$f(x) = \lim\limits_{b\to\infty} \left(\int_0^b x \cdot e^{-x^2}\, dx\right) = \lim\limits_{b\to\infty} \left(\left[-\frac{1}{2}e^{-x^2}\right]_0^b\right) = \lim\limits_{b\to\infty} \left(\left(-\frac{1}{2}e^{-b^2}\right) - \left(-\frac{1}{2}e^0\right)\right)$

$= \lim\limits_{b\to\infty} \left(-\frac{1}{2}e^{-b^2} + \frac{1}{2}\right) = \frac{1}{2}$

Aufgabe 5

$f(x) = \int_1^\infty \frac{1}{\sqrt{x}}\,dx$

$f(x) = \lim_{b\to\infty}\left(\int_1^b \frac{1}{\sqrt{x}}\,dx\right) = \lim_{b\to\infty}\left(\left[\sqrt{x}\right]_1^b\right) = \lim_{b\to\infty}\left(\left(2\sqrt{b}\right) - \left(2\sqrt{1}\right)\right)$

$= \lim_{b\to\infty}\left(2\sqrt{b} - 2\right) = \infty$

⇨ Keine endliche Fläche (kein endliches Integral)

Aufgabe 6

$f(x) = \int_0^1 \frac{1}{x^2}\,dx$

$f(x) = \lim_{b\to 0}\left(\int_b^1 \frac{1}{x^2}\,dx\right) = = \lim_{b\to 0}\left(\left[-\frac{1}{x}\right]_b^1\right) = \lim_{b\to 0}\left(\left(-\frac{1}{1}\right) - \left(-\frac{1}{b}\right)\right)$

$= \lim_{b\to 0}\left(-1 + \frac{1}{b}\right) = \infty$

⇨ Keine endliche Fläche (kein endliches Integral)